外国語科目 (数理・計算科学専攻)　　15　大修

英語　　時間 14:00 ～ 15:00

注意事項

1. 次の3問中より2問を選択し解答せよ．2問を超えて解答した場合は採点されない可能性がある．
2. 解答は1問ごとに別々の解答用紙に記入せよ．
3. 各解答用紙には必ず問題の番号および受験番号を記入せよ．

問1. 次の文章を読み，下線部の質問の意味を150字以内で説明せよ．

The question "Can one hear the shape of a drum?" is silly. Everyone knows that we hear sounds not shapes. Nevertheless this very question is the title of a famous article by Mark Kac which appeared in 1966 [9]. Not only that, the provocative question has spawned articles on the same subject with similar titles. "On hearing the shape of a drum: further results," by Stewartson and Waechter [27] and "Hearing the shape of an annular drum" by Gottlieb [4] are two examples. Moreover, other senses are becoming involved. There is the article by Pinsky [18] titled "Can you feel the shape of a manifold with Brownian motion?"

Actually, the question we are revisiting has two meanings: a mathematical one and a nonmathematical one. We describe both. As we are all aware, the sounds a drum makes when it is struck are determined by its physical characteristics, i.e., the material used, its tautness, and the size and shape. Drums vibrate at certain distinct frequencies called normal modes. The lowest or base frequency is the fundamental tone and the higher frequencies are called overtones.

The nonmathematical interpretation of the question is the following: suppose a drum is being played in one room and a person with perfect pitch, i.e., one who can identify exactly *all* the normal modes of vibration, hears but cannot see the drum. Is it possible for her to deduce the precise shape of the drum just from hearing the fundamental tone and all the overtones?

以下，数学的な意味の説明が続く．

fundamental tone　基音
overtone　上音

From "*Can one hear the shape of a drum? Revisited*" by M.H.Protter, SIAM Review, 29, 1987.

問2. 次の文章を読み, 下の (1), (2) に英語または日本語で答えよ.

Prime numbers and their properties were first studied extensively by the ancient Greek mathematicians. The mathematicians of Pythagoras's school (500 BC to 300 BC) were interested in numbers for their mystical and numerological properties. They understood the idea of primality and were interested in perfect numbers. A perfect number is one whose proper divisors – i.e., not including the number – sum to the number itself. For example, the number 6 has proper divisors 1, 2 and 3; and $1 + 2 + 3 = 6$. Therefore, 6 is a perfect number. By the time Euclid's Elements appeared in about 300 BC, several important results about primes had been proved. In Book IX of the Elements, Euclid proves that there are infinitely many prime numbers. Euclid also gives a proof of the Fundamental Theorem of Arithmetic: Every integer can be written as a product of primes in an essentially unique way. Euclid also showed that if the number $2^n - 1$ is prime then the number $2^{n-1}(2^n - 1)$ is a perfect number. The mathematician Euler – much later, in 1747 – was able to show that all even perfect numbers are of this form. It is not known to this day whether there are any odd perfect numbers.

Slightly edited excerpts from: "Prime Numbers" by J. J. O'Connor and E. F. Robertson. Available at: `www-history.mcs.st-andrews.ac.uk/history/HistTopics/Prime_numbers.html` (December 2001).

(1) (1.1) List the proper divisors of the following two numbers: 28 and 32.
  (1.2) Is 28 a perfect number? Justify your answer.
  (1.3) Is 32 a perfect number? Justify your answer.

(2) List the results concerning perfect numbers given in the text above.

問 3.　次の文章を読み，下の (1)，(2) に答えよ．

1 [ Suppose we want to describe a given object by a finite binary string. We do not care whether the object has many descriptions; however, each description should describe but one object. From among all descriptions of an object we can take the length of the shortest description as a measure of the object's complexity. It is natural to call an object "simple" if it has at least one short description, and to call it "complex" if all of its descriptions are long. ]

But now we are in danger of falling into the trap so eloquently described in 2 the Richard-Berry paradox, where we define a natural number as "the least natural number that cannot be described in less than twenty words." If this number does exist, we have just described it in thirteen words, contradicting its definitional statement. If such a number does not exist, then all natural numbers can be described in fewer than twenty words. We need to look very carefully at the notion of "description."

From "An Introduction to Kolmogorov Complexity and Its Applications—Second Edition" by Ming Li and Paul Vitányi.

(1)　括弧 1 を日本語に訳せ．

(2)　下線部 2 の "the Richard–Berry paradox" がどのようなパラドックスであるかを説明せよ．